Unconstrained nonlinear relaxations in global optimization

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The global optimization of mixed-integer nonlinear programs is an ambitious goal, with many practical applications. Here, we consider the broadest class of such problems, where no convexity or regularity assumptions are made. We only require that an expression graph representing the objective and constraints be given.

The most common approach to solve these generic problems is the *reformulation convexification* technique. It uses a symbolic reformulation to express the problem in a standard form, such as the one introduced in [1], that facilitates the implementation of various tools like convex relaxations and bound tightening techniques. Some of the most successful global solvers [2, 3] follow this approach and make use of decades of engineering in mixed-integer linear programming for robustness and efficiency.

Since the work of [4], we know that the hybridation with nonlinear programming can lead to important improvements. However, linear relaxations still rule in most global optimization solvers and remain the default setting. To show that nonlinear relaxations can be competitive, we implemented a global optimization solver that can use both linear and nonlinear relaxations. This presentation will be focused on a key idea that substantially improved the solver : the use of unconstrained nonlinear relaxations. These relaxations are obtained by augmented lagrangian duality and Wolfe duality, and have many other applications in global optimization : computing lower bounds by means of local search, robust bounds and inconsistency certificates and a new approach to solve linear programs.

References

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